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International Journal of Heat and Mass Transfer 47 (2004) 5285–5296

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Archival correlations for average heat transfer coefficients for non-circular and circular cylinders and for spheres in cross-flow

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Received 26 September 2003; received in revised form 11 June 2004

Abstract

Average Nusselt number information was collected for all textbook-standard, non-circular cylinders in cross-flow in air. These non-circular cross-sections include: squares, diamonds, flat plates perpendicular to the freestream, ellipses, hexagons, rectangles, and circles. This collection encompassed both venerable data which form the basis of the correlations recommended by current textbooks and all of the modern data that could be found in the literature. For each of the selected non-circular cross-sections, the available data were displayed, evaluated, and compared with those for all related cross-sections. On the basis of the merits of the information collected for each cross-sectional shape, correlations are recommended which are intended to form a new set of textbook standards. Also, the new correlations avoid errors made in the current textbook correlations which are caused by an inconsistency in the selection of the characteristic dimension relative to that used by the original investigators. Almost exclusively, the new correlations are based on modern experimental data. In order to provide a complete compendium of cross-flow heat transfer information, correlations for the circular cylinder and the sphere have been included in the table. The correlation for the circular cylinder is original to this paper.

Keywords: Archival correlations; Average Nusselt numbers; Circular cylinders; Non-circular cylinders; Spheres; Cross-flow

1. Introduction

A major focus of heat transfer applications involving external flows is with various objects situated in cross-flow. These objects may include cylinders of circular or non-circular cross-section, spheres, or plates. A great deal of effort has been expended on the acquisition and correlation of experimental data for the circular cylinder, and a succession of improved algebraic representations for the heat transfer characteristics has been forthcoming. On the other hand, for non-circular cylinders, relatively little modern information exists. For example, in virtually every heat transfer textbook [1–9], a table of results for a group of non-circular

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doi:10.1016/j.ijheatmasstransfer.2004.06.024

Nomenclature

C, C_{d_c}	correlation coefficients
D	diameter
d_c	equivalent diameter
d_{minor}	minor diameter of an elliptical cylinder
$\bar{h}$	surface-averaged heat transfer coefficient
k	thermal conductivity
ℓ	projected height of cylinder perpendicular to the freestream
$\overline{Nu}$	surface-averaged Nusselt number
P	cylinder perimeter
Pr	Prandtl number
Re	Reynolds number
S	facet length of a polygonal cylindrical body
U_∞	freestream velocity
Greek symbols	
μ	dynamic viscosity
ν	kinematic viscosity

cross-sections may be found which is, in fact, based on results of experiments performed in the 1920s and 30s. More recently, occasional publications dealing with non-circular cylinders in cross-flow have appeared which tend to cast doubt on the validity of those earlier results. Furthermore, as will be demonstrated here, aside from the issue of accuracy, the current textbook representations of the early data, except for [1], are somewhat in error owing to a misinterpretation of the characteristic dimension which was originally used in the correlation of the experimental data.

The aims of this paper are threefold. First, all the available data for cross-flow heat transfer over non-circular cylinders will be brought together to assess their quality and, in particular, to achieve a definitive conclusion about the doubts that have been raised about the early data. Second, and foremost, is to distill from the aforementioned collected data a new set of correlations for non-circular cylinders to serve as suggested replacements for those that are presently standard in textbooks. Lastly, the most recent and best correlations for the circular cylinder and the sphere will be added to the correlation list for non-circular cylinders in order to achieve a complete description of the available information for forced convection heat transfer in cross-flow.

The foregoing paragraphs have provided the broad context for this paper. More specific referencing will now be made in order to provide a full historical perspective for heat transfer from non-circular cylinders in cross-flow. The information to be collected here will be categorized according to whether it appeared prior to 1950 or after 1950. Any information prior to 1950 will be termed *pre-modern* while information from the later period will be termed *modern*.

The pre-modern work that has been mentioned in the foregoing is that of Reiher [10] and of Hilbert [11]. The results from these experiments were first reported in the textbook literature in the German language by Eckert. An English-language translation of that textbook appeared in 1950 [12]. The treatise by Jakob [13] was published in 1949, and it, independently, reported the same information as did Eckert. The noteworthy feature of the reported information was the characteristic dimension used in the correlation of the data. That characteristic dimension is the diameter of a circular cylinder whose perimeter is equal to that of the non-circular cylinder in question.

The 1950s was the era when modern-style heat transfer textbooks first appeared. These included those of [14–20]. In addition, newer editions of pre-modern texts such as Brown and Marco [21] and Jakob and Hawkins [22] also were published. Among these, [20] included the original information of [12,13]. On the other hand, Knudsen and Katz [18] reported the data of Reiher and of Hilbert in dimensionless form similar to that of both Eckert [12] and Jakob [13], but with one critical difference. Instead of the equivalent diameter as defined in the foregoing, Knudsen and Katz used the projected height of the object. This change of dimension introduced an error in their Nusselt–Reynolds correlation. A second error was incurred when the Reynolds number ranges cited by Jakob and Eckert were left unaltered when the projected height replaced the equivalent diameter as the characterizing dimension.

It is also interesting to note that while the first editions of many well-known textbooks such as [23–29] did not include any information for non-circular cylinders in cross-flow; some texts, such as [30], did contain non-circular correlation information. The first appearance of the errant non-circular correlations of Knudsen and Katz [18] in a first-edition of a modern heat transfer textbook seems to be in Gebhardt [31]. Thereafter, most textbooks adopted the errant correlations.

It appears that, on the basis of the foregoing historical review, that Knudsen and Katz were the first to have misinterpreted the original correlations of Hilpert and of Reiher and their English-language publication by Jakob and by Eckert. At present, the majority of textbooks include information on heat transfer for non-circular cylinders in cross-flow [1–9]. With the exception of [1], the information is reported in the incorrect form originated by Knudsen and Katz.

Modern data for cross-flow heat transfer for non-circular objects have been collected by [32–43]. For circular cylinders in cross-flow, correlations of modern data have been provided by [44–48], while for spheres in cross-flow, the correlating equation developed by Whitaker [47] appears to be the most widely accepted source of information.

2. Knowledgebase for heat transfer from non-circular cylinders in cross-flow

The defining feature of the fluid flow about non-streamlined objects in cross-flow is illustrated in Fig. 1, which, for concreteness, is a cylinder of square cross-section. As seen there, except at very low Reynolds numbers, the fluid is not able to follow the contour of the downstream portion of the object and separates from it. As a consequence, a wake forms on the leeward side of the object as depicted in the diagram. The nature of the pattern of fluid flow in the wake depends decisively on the shape of the object as well as on the Reynolds number. Over a sufficiently large range of Reynolds numbers, the relative contributions to the overall rate of heat transfer from the unseparated and separated regions of the flow evolve as a function of the Reynolds number. For lower Reynolds numbers, the contribution from the unseparated region dominates, while at higher Reynolds numbers, the opposite situation prevails. This behavior is readily identified when the heat transfer data cover a range of many decades in the value of the Reynolds number. On the other hand, when the available data are more or less confined to a decade variation of the Reynolds number, it is difficult to identify this behavior.

For the case of the circular cylinder in cross-flow, the available data are so extensive and cover such a large range of Reynolds numbers that the evolution of the changing contributions of the two flow regimes is readily observable. On the other hand, the modern data for non-circular cylinders in cross-flow are confined to a Reynolds number range that spans values slightly greater than a decade. As a consequence, the heat transfer correlations for non-circular cylinders are generally of a simpler form than those that are encountered for the circular cylinder. In particular, all of the available information for non-circular cylinders is in the form of a power-law relationship between the Nusselt and Reynolds numbers.

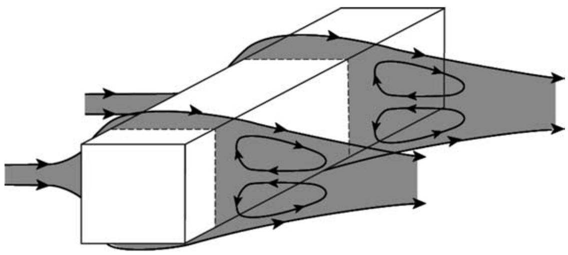


Fig. 1. Fluid flow over a square cylinder whose axis is oriented perpendicular to the flow.

The various cross-sections of the non-circular cylinders include the square, diamond, flat plate oriented perpendicular to the flow, ellipse, hexagon, and rectangle. To complete the compilation for cross-flow heat transfer over the entire gambit of commonly encountered objects, results will also be conveyed here for the circular cylinder itself as well as for the sphere.

2.1. Collection of information for cross-flow heat transfer

2.1.1. Information from the pre-modern era

The sources of pre-modern information on cross-flow heat transfer for the non-circular cylinders are the experiments of [10,11] which were first reported independently in the English-language literature by Eckert [12] and Jakob [13]. The cross-sectional shapes of the reported data included the square, diamond, hexagon, ellipse, and the flat plate oriented perpendicular to the flow. The experimental data were originally correlated by making use of an equivalent diameter d_c for a circular cylinder. The diameter d_c was chosen so that the perimeter of the non-circular cross-section is the same as that of the equivalent circular cylinder.

The defining equation for d_c follows as

$$\pi d_c = P \quad \text{or} \quad d_c = P/\pi \tag{1}$$

where P is the perimeter of the cross-section of the non-circular cylinder.

With this definition, the available heat transfer results, all for air as the participating fluid, were correlated in the form

$$\overline{Nu}_{d_c} = C_{d_c} Re_{d_c}^n \tag{2}$$

where

$$\overline{Nu}_{d_c} = \frac{\bar{h} d_c}{k}, \quad Re_{d_c} = \frac{U_\infty d_c}{\nu} \tag{3}$$

The numerical values of C_{d_c} and n, as listed in Eckert [12] and in Jakob [13], are specific to the geometries of the particular non-circular cylinders that were investigated.

For convenience in applications, it has become conventional to use the projected height ℓ of the cross-flow object as the characteristic dimension in the Nusselt–Reynolds correlations instead of the equivalent diameter d_c. If the correlating equation, Eq. (2), is rephrased with ℓ as the characteristic dimension, there results

$$\overline{Nu}_\ell = C \cdot Re_\ell^n \tag{4}$$

where

$$C = C_{d_c} \left(\frac{\ell}{d_c}\right)^{1-n} \tag{5}$$

The ratio ℓ/d_c has been evaluated here for all of the cross-sectional shapes that were tabulated in Eckert [12] and in Jakob [13]. The original Hilpert–Reiher correlations, rephrased as in Eq. (4), are listed in Table 1. That information is tabulated under the heading "Correct Values". The form of the Hilpert–Reiher correlations which appears in the current heat transfer textbooks all include the Knudsen–Katz error. Those correlations are listed in the table under the heading "Incorrect Values". With regard to the latter, it is important to note that in some of the textbooks, a multiplicative factor $Pr^{1/3}$ had been appended to the correlation with the view toward endowing it with a more general utility.

This appendage, while laudatory in intent, is somewhat arbitrary inasmuch as the original data were collected specifically for air. The present authors have reversed the $Pr^{1/3}$ appendage and restored the textbook correlations to be applicable only to air.

Inspection of the table reveals two types of errors in the current textbook correlations. First, the multiplicative constant C displays errors up to 27%, depending on the specific geometry. The second error is in the range of the Reynolds number for which the correlations are applicable. The extent of the latter error is up to 83%, once again, depending on the specific cross-sectional shape.

2.1.2. Information from the modern era

The presentation of information that follows will be categorized with regard to the specific cross-sectional shapes for which data are presently available.

2.1.2.1. Square cylinders.

The orientation of the square cylinder with respect to the freestream flow is illustrated as the first entry in the left-hand column of Table 1.

The available surface-averaged Nusselt number information for cross-flow over a cylinder of square cross-section has been brought together in Fig. 2. The figure includes the pre-modern results of Reiher and of Hilpert as well as the modern results of Igarashi [32], Ahmed [33], and Goldstein [36]. Although the displayed information spans about 70 years, it is satisfying to note that aside from the Ahmed results, the others display slopes which are virtually identical. There is, however, a considerable spread in the magnitude of the Nusselt numbers. The largest spread is between the Hilpert data

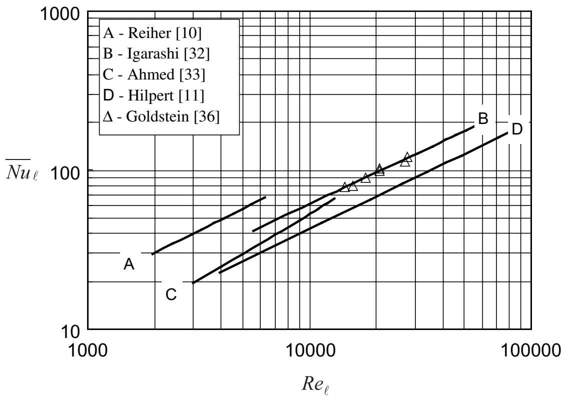
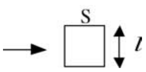
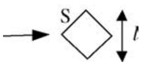
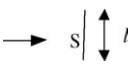
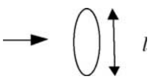
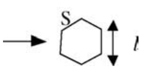
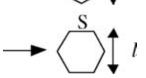


Fig. 2. Collected average Nusselt numbers for cross-flow heat transfer in air for a cylinder of square cross-section.

Table 1
Comparison of the original Hilpert–Reiher [10,11] correlations with that of current heat transfer textbooks (the correlations are for air flow)

Cylinder cross-section	Correct values		Incorrect values		n
	C	$Re_\ell \times 10^{-3}$	C	$Re_\ell \times 10^{-3}$	
	0.149 0.085	2.0–6.3 3.9–79	0.160 0.092	2.5–8 5–100	0.699 0.675
	0.272 0.232	2.8–8.3 5.6–111	0.261 0.222	2.5–7.5 5–100	0.624 0.588
	0.231	6.3–23.6	0.205	4–15	0.731
	0.177	1.4–8.2	0.224	2.5–15	0.612
	0.090	4.1–20.5	0.085	3.0–15	0.804
	0.146 0.035	5.2–20.4 20.4–105	0.144 0.035	5–19.5 19.5–100	0.638 0.782
	0.133	4.5–90.7	0.138	5–100	0.638

and the Reiher data, both pre-modern. On the other hand, the Igarashi and Goldstein data, both modern, are highly reinforcing. Further reinforcement stems from the fact that the Igarashi experiment was for heat transfer while the Goldstein work was performed for mass transfer (naphthalene sublimation technique). Additional reinforcement is provided by the fact that the displayed Igarashi results actually are the merging of three investigations by that author.

The fact that the Igarashi results lie somewhat higher than those of Goldstein is physically reasonable in view of the boundary conditions that were in force in the two experiments. In the case of Igarashi, the uniform heat flux boundary condition was implemented while in Goldstein's experiments, the employed naphthalene sublimation technique corresponds, by analogy, to heat transfer in the presence of uniform surface temperature. It is well established that the uniform heat flux boundary condition always gives rise to higher Nusselt numbers than does the uniform surface temperature condition.

It is the considered opinion of the present authors that the best current representation of the average Nusselt number results for a square cylinder in cross-flow is

$$\overline{Nu}_\ell = 0.14 \cdot Re_\ell^{0.66}, \quad 5000 \leqslant Re_\ell \leqslant 60{,}000 \tag{6}$$

which is Igarashi's correlation.

The curve in Fig. 2 attributed to Ahmed presents an enigma. In point of fact, the displayed curve was extracted from the experimental data themselves by the present authors and was not taken from the correlations provided in the Ahmed paper. Those correlations could not be numerically evaluated because they depend on information that was internal to that investigation. The plotted curve in Fig. 2 has a slope of approximately 0.83. Such a slope is not consonant with the nature of the physical processes that are believed to be occurring. Those processes encompass laminar boundary-layer flow on the forward-facing surface and separated flow on the other surfaces. The corresponding Reynolds-number dependences of these flows are, respectively, the 1/2 power and the 2/3 power. Consequently, it is believed that the 0.83 power is unreasonable.

2.1.2.2. Diamond cylinders. The diamond cylinder in cross-flow is pictured as the second entry in the left-hand column of Table 1. The diamond configuration is, in truth, the square cylinder rotated by 45° so that one of the corners thrusts forward.

The available information for the average Nusselt number for the diamond cylinder in cross-flow has been brought together in Fig. 3. Two of the lines, A and C, in this figure correspond to the pre-modern data of [10,11], respectively. The modern data is that of Igarashi [32] and of Yoo [35]. As in the case of the square cylinder in cross-flow, Igarashi's data correspond to the uniform heat flux boundary condition while Yoo's data, obtained

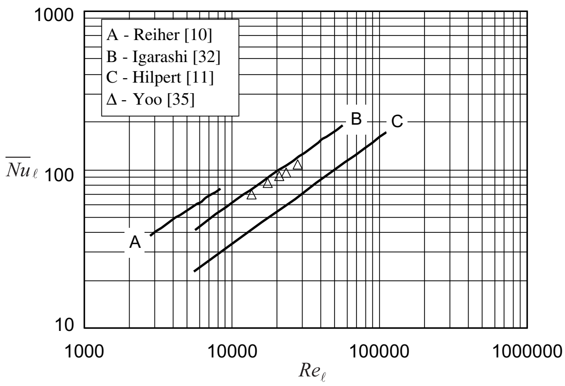


Fig. 3. Collected average Nusselt numbers for cross-flow heat transfer in air for a cylinder of diamond cross-section.

via the naphthalene sublimation technique, correspond to heat transfer for the uniform wall temperature boundary condition.

It is clear from the figure that all of the data, which span approximately 70 years, are characterized by slopes that are virtually identical. With regard to the magnitudes of the Nusselt numbers from the different sources, the Yoo data support those of Igarashi, both sets of data being in the modern category. On the other hand, the pre-modern data of Hilpert and of Reiher are separated by about 40%. In recognition of the greater resolving power of modern instrumentation and of the mutual support of [32,35], the recommended correlation for the average Nusselt number for the diamond cylinder in cross-flow is

$$\overline{Nu}_\ell = 0.27 \cdot Re_\ell^{0.59}, \quad 6000 \leqslant Re_\ell \leqslant 60{,}000 \tag{7}$$

It is interesting and relevant to compare the heat transfer capabilities of the square and diamond cylinders in cross-flow. The most meaningful comparison is that involving the heat transfer coefficients $\overline{h}_{\text{square}}$ and $\overline{h}_{\text{diamond}}$. For the former, the projected height ℓ which appears in both the Nusselt and Reynolds numbers is equal to the side S of the square. On the other hand, in the case of the latter, $\ell = \sqrt{2}S$. Since S is common to both the square and the diamond, it is natural to make the comparison of the heat transfer coefficients for the same value of S. When the characteristic length ℓ which appears in Eqs. (6) and (7) is replaced by S and $\sqrt{2}S$, respectively, there results

$$\frac{\overline{h}_{\text{square}}}{\overline{h}_{\text{diamond}}} = 0.6 \cdot \left(\frac{U_\infty S}{\nu}\right)^{0.07}, \quad 5000 \leqslant \frac{U_\infty S}{\nu} \leqslant 42{,}000 \tag{8}$$

To continue the comparison, Eq. (8) may be evaluated at the lower and upper Reynolds numbers of the stated range of validity. The evaluation gives

$$\frac{\bar{h}_{\text{square}}}{\bar{h}_{\text{diamond}}} = 1.09, \quad \frac{U_\infty S}{v} = 5000 \tag{9}$$

and

$$\frac{\bar{h}_{\text{square}}}{\bar{h}_{\text{diamond}}} = 1.26, \quad \frac{U_\infty S}{v} = 42,000 \tag{10}$$

These equations indicate that the square orientation is a more capable heat transfer configuration than is the diamond orientation. As a justification for this behavior, it may be noted that in the case of the square, three of the four sides are washed by separated regions while for the diamond, only two of the faces experience flow separation. Inasmuch as separated-region heat transfer is energized at the higher Reynolds numbers, as is in accord with Eqs. (9) and (10), it is believed that the separated-region explanation is valid.

2.1.2.3. Flat plates perpendicular to the freestream. A schematic diagram of a flat plate situated perpendicular to an oncoming uniform freestream is pictured as the third entry in the left-hand column of Table 1. The information given in the table corresponds to the average Nusselt number for simultaneous heat transfer at both the front-facing and rear-facing surfaces of the plate. In contrast, the modern data apply separately to either the front face or the rear face of the plate, respectively Sparrow [38] and Igarashi [37].

Figure 4 displays the available heat transfer information. The uppermost line, A, in the figure is the pre-modern data of Reiher [10] which corresponds to heat transfer from both faces of the plate. Lines B and D are, respectively, for rear-face and front-face heat transfer. The separate data from B and D have been combined by the present authors to yield an estimate of the simultaneous heat transfer for both faces of the plate. Curve C represents the resulting correlation for the situation of simultaneous heat transfer.

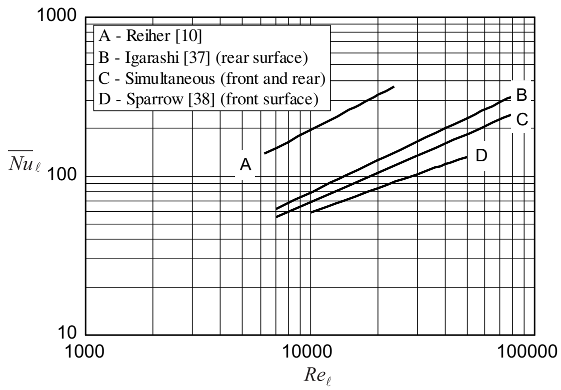


Fig. 4. Collected average Nusselt numbers for cross-flow heat transfer in air from a flat plate oriented perpendicular to a freestream.

Inspection of the figure indicates that the pre-modern data markedly overestimate the heat transfer. The results denoted as B and D are mutually supportive from two standpoints. First, the front-face data have a lesser slope than that of the rear-face data. This is altogether reasonable because the front face is washed by a laminar boundary layer while the rear face is in a separated region. It is well established that the Reynolds number dependence for the latter corresponds to a larger exponent than that for the former. Second, the curves B and D diverge with increasing Reynolds number, as they must according to their different exponents. It is also clear that, with decreasing values of the Reynolds number, the two lines would cross, so that the front-face heat transfer would exceed that of the rear-face heat transfer. This behavior has been noted in the past for the intensively studied case of the circular cylinder in cross-flow.

On the basis of the preceding discussion, the following equations are offered for heat transfer at the front face only, at the rear face only, and for the simultaneous heat transfer at both faces.

$$\overline{Nu}_\ell = 0.592 \cdot Re_\ell^{1/2}, \quad 10,000 \leqslant Re_\ell \leqslant 50,000, \quad \text{front surface} \tag{11}$$

$$\overline{Nu}_\ell = 0.17 \cdot Re_\ell^{2/3}, \quad 7000 \leqslant Re_\ell \leqslant 80,000, \quad \text{rear surface} \tag{12}$$

$$\overline{Nu}_\ell = 0.25 \cdot Re_\ell^{0.61}, \quad 10,000 \leqslant Re_\ell \leqslant 50,000, \quad \text{both surfaces} \tag{13}$$

Note that the front-surface Nusselt number is proportional to the 1/2 power of the Reynolds number whereas the rear-surface proportionality is to the 2/3 power. These findings are in excellent accord with previous experience with blunt objects in cross-flow.

2.1.2.4. Elliptical cylinders. The elliptic cylinders pictured in the first column of Table 1 correspond to two cross-flow orientations. In the first, the cylinder is oriented so that its long axis is parallel to the direction of the freestream flow. In the second, the long axis is perpendicular to the freestream. Both of these ellipses appear to have the same ratio of major to minor axes (aspect ratio), although that ratio is unspecified. Although the present authors have measured the aspect ratio to be 2.5:1, as it appears in the original drawings of Eckert [12] and of Jakob [13], there is still some uncertainty as to whether the measured ratio is, in fact, the true ratio. In the absence of other information, it will be assumed that the data are for a 2.5:1 ellipse.

The aforementioned pre-modern data are brought together with the modern data of Ota [41] and Kondjoyan [42] in Fig. 5. The information conveyed by the figure naturally subdivides itself into two groupings. The upper pair of curves (A and B) both correspond to the situation

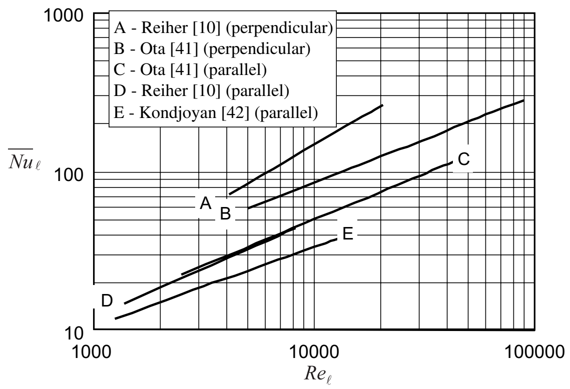


Fig. 5. Collected average Nusselt numbers for cross-flow heat transfer in air from ellipses oriented either perpendicular or parallel to a freestream.

in which the long axis of the ellipse is perpendicular to the direction of the oncoming flow. The aspect ratios for these two ellipses are 2.5:1 and 2:1, respectively for the upper and lower of the two curves. Results for the cases in which the long axis of the cylinder is parallel to the flow direction lie in the lower part of the figure. The upper pair of these curves correspond respectively to aspect ratios of 2:1 and 2.5:1 (Curves C, D, and E). The lowermost line in this grouping corresponds to an aspect ratio of 4:1.

It is clear from the results presented in Fig. 5 that the Nusselt numbers for the case in which the flow is perpendicular to the long axis are greater than those for the case in which the flow is parallel to the long axis. It will now be shown that the same relationship holds for the actual heat transfer coefficients. For this purpose, the results of Ota will be used for the comparison. For the case in which the long axis is perpendicular to the flow, the Ota correlation is

$$\overline{Nu}_\ell = 0.566 \cdot Re_\ell^{0.545}, \quad 5000 \leqslant Re_\ell \leqslant 90,000, \quad \text{perpendicular} \tag{14}$$

On the other hand, for the case in which the long axis is parallel to the flow, Ota gives

$$\overline{Nu}_\ell = 0.256 \cdot Re_\ell^{0.573}, \quad 2500 \leqslant Re_\ell \leqslant 45,000, \quad \text{parallel} \tag{15}$$

For the purpose of comparing the values of the heat transfer coefficients, a common dimension must be used in the Nusselt and the Reynolds numbers. That dimension will be selected to be the minor axis, d_{minor}, of the ellipse. If note is taken of the fact that the aspect ratios of the two cases in question are 2:1, it follows that

$$\frac{\overline{h}_{\text{perpendicular}}}{\overline{h}_{\text{parallel}}} = 1.61\left(\frac{U_\infty \cdot d_{\text{minor}}}{v}\right)^{-0.028}, \quad 5000 \leqslant \frac{U_\infty \cdot d_{\text{minor}}}{v} \leqslant 45,000 \tag{16}$$

When the foregoing ratio is evaluated at the extremes of the Reynolds number range, there is obtained

$$\frac{\overline{h}_{\text{perpendicular}}}{\overline{h}_{\text{parallel}}} = 1.27, \quad \frac{U_\infty \cdot d_{\text{minor}}}{v} = 5000 \tag{17}$$

and

$$\frac{\overline{h}_{\text{perpendicular}}}{\overline{h}_{\text{parallel}}} = 1.19, \quad \frac{U_\infty \cdot d_{\text{minor}}}{v} = 45,000 \tag{18}$$

which reaffirms the primacy of the perpendicular orientation.

Although the foregoing comparison is a clear demonstration that the perpendicular orientation is to be favored if higher rates of heat transfer are desired, the correlations conveyed by Eqs. (14) and (15) present a puzzlement. Based on the fact that the perpendicular orientation is subject to a large separated region on its rear-facing surface, whereas the parallel orientation experiences a much smaller separated region, it would be expected that the dominance of the separated region for the former would give rise to a higher power-law dependence. It is, therefore, difficult to rationalize the fact that the power-law for the perpendicular orientation is 0.545 while that for the parallel orientation is 0.573. This seeming disparity does not have a ready explanation.

It remains to select recommended correlations from among those that are displayed in Fig. 5. Both Ota and Reiher provide candidate correlations for both the perpendicular and parallel orientations. With regard to Reiher, the aspect ratio of his elliptical cylinder is still in doubt. This fact, coupled with the already stated judgment that when comparing the relative merits of a premodern versus a modern investigation, the latter is to be preferred when everything else is equal. By this reasoning, the Ota correlations given by Eqs. (14) and (15) for a 2:1 aspect-ratio ellipse are recommended for archival consideration.

Still remaining for evaluation is Curve E of Fig. 5. This curve corresponds to the case of an ellipse with a 4:1 aspect ratio aligned parallel to the flow. The fact that Curve E lies below its orientation counterpart represented by Curve C is reasonable when viewed with respect to the facts that the two cases in question have different aspect ratios. The longer ellipse (4:1 aspect ratio) is represented by Curve E. Prior experience has demonstrated that objects that are elongated in the streamwise direction are characterized by lower Nusselt numbers.

2.1.2.5. Hexagonal cylinders. The presentation of results continues with the case of hexagonal cylinders oriented perpendicular to the oncoming stream. As seen in the left-hand column of Table 1, there are two specific orientations that can be considered. One of these is characterized by a freestream which is perpendicular to one of the flat facets of the cylinder while the other case corresponds to the freestream impinging on the forward-most apex. An intensive literature search failed to unearth any modern data for either of the hexagonal cases pictured in Table 1, perhaps reflecting the infrequent use of hexagonal cylinders.

The data to be presented, then, is that of Hilpert [11], the correlating equations for which are

$$\overline{Nu}_\ell = 0.146 \cdot Re_\ell^{0.638},\quad 5200 \leqslant Re_\ell \leqslant 20,400, \quad \text{flat facet forward} \tag{19}$$

$$\overline{Nu}_\ell = 0.035 \cdot Re_\ell^{0.782},\quad 20,400 \leqslant Re_\ell \leqslant 105,000, \quad \text{flat facet forward} \tag{20}$$

$$\overline{Nu}_\ell = 0.133 \cdot Re_\ell^{0.638},\quad 4500 \leqslant Re_\ell \leqslant 90,700, \quad \text{apex facet forward} \tag{21}$$

These equations are displayed graphically in Fig. 6. It is seen from the figure that the flat-facet-forward orientation provides larger values of the Nusselt number than does the apex-forward orientation. The relative merits of the two orientations must, however, be compared on the basis of the respective heat transfer coefficients rather than on the basis of the respective Nusselt numbers. If S denotes the length of a facet of the hexagon, then the transformation of Eqs. (20) and (21) to include S-dependent parameters leads to

$$\frac{\bar{h}_{\text{flat}}}{\bar{h}_{\text{apex}}} = 0.276\left(\frac{U_\infty S}{\nu}\right)^{0.144}, \quad 10,200 \leqslant \left(\frac{U_\infty S}{\nu}\right) \leqslant 52,400 \tag{22}$$

If this ratio is evaluated at the upper and lower Reynolds numbers of the range indicated in Eq. (22), there follows

$$\frac{\bar{h}_{\text{flat}}}{\bar{h}_{\text{apex}}} = 1.04, \quad \left(\frac{U_\infty S}{\nu}\right) = 10,200 \tag{23}$$

$$\frac{\bar{h}_{\text{flat}}}{\bar{h}_{\text{apex}}} = 1.32, \quad \left(\frac{U_\infty S}{\nu}\right) = 52,400 \tag{24}$$

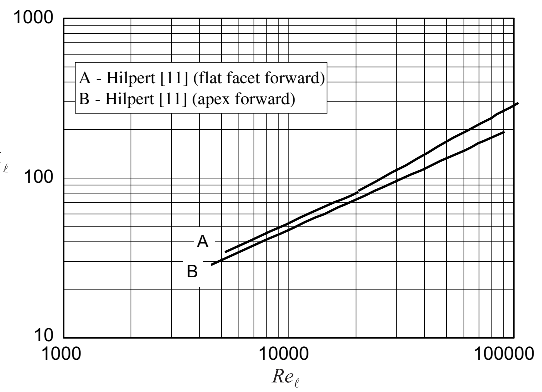


Fig. 6. Average Nusselt numbers for cross-flow heat transfer in air for a cylinder of hexagonal cross-section from Hilpert [11].

The ratios displayed in Eqs. (23) and (24) indicate that if there is a choice of orientation to maximize the rate of heat transfer, it is appropriate to select the flat-facet-forward case. The explanation for this behavior may be attributed to the expected differences in the flow patterns for the two orientations. For the flat-facet-forward orientation, it may be anticipated that five of the six facets may be either fully or partially washed by a recirculation zone. On the other hand, for the apex-forward case, only four of the six faces are expected to experience flow separation. In the Reynolds number range of the present data, the presence of recirculation is believed to be enhancing with regard to the rate of heat transfer. Consequently, the greater extent of the regions of recirculation for the flat-facet-forward orientation is responsible for its primacy with regard to heat transfer.

2.1.2.6. Rectangular cylinder. Experimental data for cylindrical objects of rectangular cross-section have been collected by Igarashi [40,43]. The aspect ratios, defined as the streamwise length to the height perpendicular to the freestream, that were investigated ranged from 0.2 to 1.5. The data were presented in terms of Nusselt–Reynolds power-law representations and are plotted in Fig. 7. The correlations for aspect ratios ranging between 0.67 and 1.5 display the 2/3-power Reynolds number dependence that is characteristic of separated flows, while the Reynolds-number dependences for the lower-aspect ratios of 0.2 and 0.33 have a lower exponent.

There are some reservations which should be considered with respect to these data that merit further consideration. One of these issues is the possible blockage of the wind tunnel cross-section for the cases where the aspect ratio is considerably less than 1. For example, for the aspect ratio of 0.2, the object occupied 1/8 of the wind-tunnel cross-section. A second issue has to do with

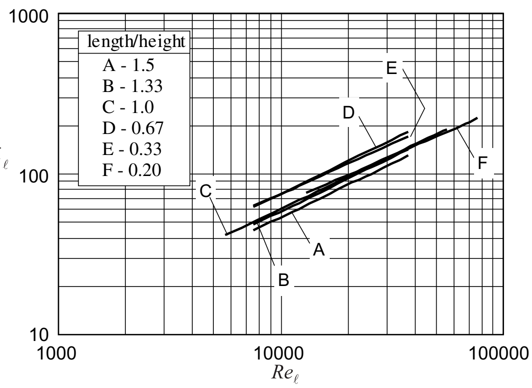


Fig. 7. Average Nusselt numbers for cross-flow heat transfer in air for cylinders of rectangular cross-sections of varying aspect ratios from Igarashi [40] and [43].

the end effects due to the interactions of the test object and the wind-tunnel sidewalls. In the extreme case, the height of the object was 1/3 of the spanwise width of the wind tunnel, whereas in the other cases, this ratio was 1/5. Experience suggests that a smaller value of the ratio is needed to ensure the absence of end effects.

The foregoing uncertainties aside, the Igarashi data presented in Fig. 7 appear to be the only available information for cylinders of rectangular cross-section in cross-flow. Therefore, pending the collection of new data, the Igarashi correlations are recommended.

$$\overline{Nu}_\ell = 0.26 \cdot Re_\ell^{0.60}, \quad 13,000 \leqslant Re_\ell \leqslant 77,000, \ \text{aspect ratio} = 0.2 \tag{25}$$

$$\overline{Nu}_\ell = 0.25 \cdot Re_\ell^{0.62}, \quad 7500 \leqslant Re_\ell \leqslant 37,500, \ \text{aspect ratio} = 0.33 \tag{26}$$

$$\overline{Nu}_\ell = C \cdot Re_\ell^{0.667}, \quad 7500 \leqslant Re_\ell \leqslant 37,500, \ \text{aspect ratios} \geqslant 0.67 \tag{27}$$

where $C = 0.163$, 0.14, 0.127, 0.116 for aspect ratios of 0.67, 1.0, 1.33, and 1.5, respectively.

3. Knowledgebase for heat transfer from circular cylinders and spheres in cross-flow

3.1. Circular cylinders

It has already been noted that heat transfer from a circular cylinder in cross-flow has evoked the most extensive experimental study among all external heat transfer situations. Correspondingly, there have been a number of proposed correlations to bring the data together in the most useful manner for practice. The correlations have been constructed using two different bases which, respectively, relate to the manner in which the fluid properties are evaluated. One set of correlations is based on the use of the so-called "film temperature", which is the average of the freestream and surface temperatures. The other set makes use of the freestream temperature for evaluating the fluid properties. Among the former are the correlations of Morgan [44] and of Churchill and Bernstein [45]. For the latter, the best known correlations are of Zhukauskas [46,48] and of Whitaker [47]. Of these, the correlations of Morgan, Churchill, and Zhukauskas are provided as piecewise algebraic equations, with each piece being assigned a Reynolds number range. On the other hand, Whitaker provides a single algebraic expression.

The present authors have reviewed the aforementioned correlations and have concluded that there is a need for improvement from the standpoint of ease of practical calculation. First, a continuous representation is preferred over a piecewise representation. Second, the use of properties evaluated at the freestream appears to offer a simplification in that the freestream conditions are normally known a priori. For these reasons, a new correlation is offered here.

An examination of the Zhukauskas correlation indicates that the low-Reynolds number tail of his representation tends to overpredict the Nusselt number whereas that of Whitaker tends to underpredict in that range of Reynolds numbers. For these same Reynolds numbers, the new correlation to be put forth here favors gases for Prandtl numbers of the order of 1 and favors liquids for higher Prandtl numbers. This tendency results in a more appropriate representation. For higher Reynolds numbers, the new correlation lies between the predictions of Zhukauskas and of Whitaker.

The new correlation is

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.25 + (0.4Re_D^{1/2} + 0.06Re_D^{2/3})Pr^{0.37}\left(\frac{\mu}{\mu_{\text{wall}}}\right)^{1/4}, \quad 1 \leqslant Re_D \leqslant 10^5 \tag{28}$$

In this equation, all of the fluid properties except μ_{wall} are to be evaluated at the freestream temperature. With regard to μ_{wall}, it is to be evaluated at the temperature of the bounding surface.

3.2. Spheres

Among the available correlations for the average Nusselt number for a sphere in a uniform freestream, that of Whitaker is widely regarded as being the most accurate as well as being very convenient to use in practice. The algebraic representation provided by Whitaker is

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 2 + (0.4Re_D^{1/2} + 0.06Re_D^{2/3})Pr^{0.4}\left(\frac{\mu}{\mu_{\text{wall}}}\right)^{1/4}, \quad 1 \leqslant Re_D \leqslant 10^5 \tag{29}$$

Here, as with Eq. (28), the fluid properties are to be evaluated at the freestream temperature, except for μ_{wall}, which is to be evaluated at the surface temperature.

4. Comparison of results for hexagonal and circular cylinders in cross-flow

As previously noted, the authors were not able to unearth any modern data for cross-flow heat transfer from hexagonal cylinders. On the other hand, a hexagonal cross-section can be regarded as nearly circular in shape. With this in mind, the pre-modern results of Hilpert [11] for hexagonal cylinders are brought together in Fig. 8 with the modern correlation for circular cylinder. The particular circular-cylinder correlation chosen for the comparison is that of Churchill and Bernstein [45]. This choice was motivated by the fact that both Hilpert and

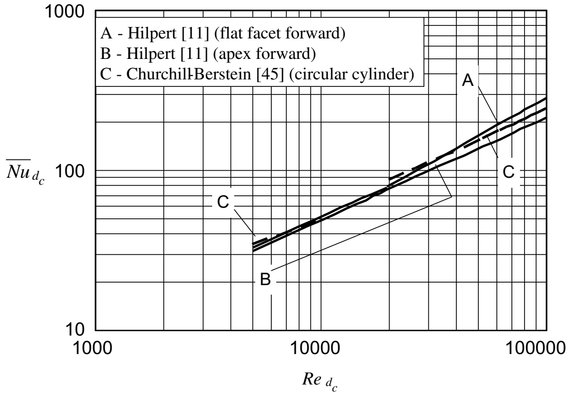


Fig. 8. Comparison of averaged Nusselt results for air for hexagonal and circular cylinders in cross-flow.

Churchill–Bernstein used fluid properties in their correlations that were evaluated at the film temperature.

For a meaningful comparison, the characteristic dimension in the hexagonal cylinder results is the equivalent diameter originally used by Hilpert. That diameter was defined so that the perimeter of the actual hexagon was matched with the perimeter of an equivalent circle.

Inspection of Fig. 8 reveals excellent agreement between the pre-modern hexagonal data and the modern circular results. This level of agreement adds support for the validity of the hexagonal results and establishes them as a basis for application.

5. Summary tabulation of correlations for the average Nusselt numbers for objects in cross-flow in air

The correlations for the various cross-sectional shapes have been collected and are presented in Table 2. The table includes the Reynolds number ranges over which the respective correlations are valid.

6. Concluding remarks

This paper has sought to provide definitive information for surface-averaged Nusselt numbers for all textbook-standard bodies in cross-flow in air. Information was collected both from the pre-modern era (prior to 1950) and the modern era (from 1950 to the present). The objects considered included cylinders whose cross-sections are: (1) square, (2) diamond, (3) flat plate perpendicular to the freestream, (4) elliptical, (5) hexagonal, (6) rectangular, and (7) circular. In addition, the best available information for a (8) sphere in a uniform flow is reported. In particular, for each of the selected non-circular cross-sections, the available data were

Table 2
Summary of correlations for convective heat transfer results for air flow over blunt objects

Shape	Correlation	Range
Square	$\overline{Nu}_\ell = 0.14 \cdot Re_\ell^{0.66}$	$5000 \leqslant Re_\ell \leqslant 60{,}000$
Diamond	$\overline{Nu}_\ell = 0.27 \cdot Re_\ell^{0.59}$	$6000 \leqslant Re_\ell \leqslant 60{,}000$
Flat plate perpendicular to freestream (front surface)	$\overline{Nu}_\ell = 0.592 \cdot Re_\ell^{1/2}$	$10{,}000 \leqslant Re_\ell \leqslant 50{,}000$
Flat plate perpendicular to freestream (rear surface)	$\overline{Nu}_\ell = 0.17 \cdot Re_\ell^{2/3}$	$7000 \leqslant Re_\ell \leqslant 80{,}000$
Flat plate perpendicular to freestream (both surfaces)	$\overline{Nu}_\ell = 0.25 \cdot Re_\ell^{0.61}$	$10{,}000 \leqslant Re_\ell \leqslant 50{,}000$
Ellipse (perpendicular to Freestream, aspect ratio 2:1)	$\overline{Nu}_\ell = 0.566 \cdot Re_\ell^{0.545}$	$5000 \leqslant Re_\ell \leqslant 90{,}000$
Ellipse (parallel to freestream, aspect ratio 2:1)	$\overline{Nu}_\ell = 0.256 \cdot Re_\ell^{0.573}$	$2500 \leqslant Re_\ell \leqslant 45{,}000$
Hexagon (flat surface forward)	$\overline{Nu}_\ell = 0.146 \cdot Re_\ell^{0.638}$	$5200 \leqslant Re_\ell \leqslant 20{,}400$
	$\overline{Nu}_\ell = 0.035 \cdot Re_\ell^{0.782}$	$20{,}400 \leqslant Re_\ell \leqslant 105{,}000$
Hexagon (apex forward)	$\overline{Nu}_\ell = 0.133 \cdot Re_\ell^{0.638}$	$4500 \leqslant Re_\ell \leqslant 90{,}700$
Rectangle (aspect ratio = 0.2)	$\overline{Nu}_\ell = 0.26 \cdot Re_\ell^{0.60}$	$13{,}000 \leqslant Re_\ell \leqslant 77{,}000$
Rectangle (aspect ratio = 0.33)	$\overline{Nu}_\ell = 0.25 \cdot Re_\ell^{0.62}$	$7500 \leqslant Re_\ell \leqslant 37{,}500$
Rectangle (aspect ratio = 0.67)	$\overline{Nu}_\ell = 0.163 \cdot Re_\ell^{0.667}$	$7500 \leqslant Re_\ell \leqslant 37{,}500$
Rectangle (aspect ratio = 1.33)	$\overline{Nu}_\ell = 0.127 \cdot Re_\ell^{0.667}$	$7500 \leqslant Re_\ell \leqslant 37{,}500$
Rectangle (aspect ratio = 1.5)	$\overline{Nu}_\ell = 0.116 \cdot Re_\ell^{0.667}$	$7500 \leqslant Re_\ell \leqslant 37{,}500$
Circle	$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.25 + (0.4Re_D^{1/2} + 0.06Re_D^{2/3})Pr^{0.37}\left(\frac{\mu}{\mu_{\mathrm{wall}}}\right)^{1/4}$	$1 \leqslant Re_D \leqslant 10^5$
Sphere	$\overline{Nu}_D = \frac{\bar{h}D}{k} = 2 + (0.4Re_D^{1/2} + 0.06Re_D^{2/3})Pr^{0.4}\left(\frac{\mu}{\mu_{\mathrm{wall}}}\right)^{1/4}$	$1 \leqslant Re_D \leqslant 10^5$

displayed, evaluated, and compared with those for all related cross-sections.

A special feature of the comparisons was the assessment of the validity of the pre-modern information. That information is reported in virtually all of the current heat transfer textbooks. Furthermore, with regard to the reporting of the pre-modern information, it is noted that all current textbook renditions are somewhat in error in that the characteristic dimension used is not consistent with the characteristic dimension of the original correlations. The errors associated with this inconsistency were found to be appreciable for certain of the non-circular cylindrical cross-sections.

In general, from the totality of collected information, recommendations were made which are intended to serve as an archival set of correlations for non-circular cylinders in cross-flow. These correlations are listed in Table 2. Aside from the case of the hexagonal cross-section, all the correlations are based on modern data, in contradistinction to the pre-modern data which presently form the basis of the correlations conveyed in current heat transfer textbooks. For the hexagonal cross-section, no modern data were found in the literature, so that the pre-modern correlations are recommended as a default. However, comparisons between results for the hexagonal cross-section and the circular cross-section revealed mutually reinforcing behavior, thereby lending support to the pre-modern hexagonal information.

In order to provide a complete compendium of cross-flow heat transfer information for air flow, correlations for the circular cylinder and the sphere have been included in the table. The correlation for the circular cylinder is original to this paper.

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